DEFINITE DESCRIPTIONS IN FREGE’S THEORY OF ARITHMETIC AND LOGICAL ANALYSIS OF A PRIORI INTUITIONS IN MATHEMATICS

Özge Ekin
Bogazici University - Freie University
Philosophy Department
Istanbul - Berlin
For the source of its axioms, geometry rests on intuition and arithmetical objects are not intuitive and fundamental rules of arithmetic cannot be based on intuition.
Frege’s Thesis

1) Always to separate sharply the psychological from the logical, the subjective from the objective;

2) Never to ask for the meaning of a word in isolation but only in the context of a proposition;

3) Never to lose sight of the distinction between concept (Begriff) and object (Gegenstand).

(cf. FA, p. X)
The cardinal number that belongs to the concept $F$ is the extension of the concept “equinumerous to the concept $F$” [For some concept $F$]

(FA, §68)

1. The same number belongs to the concept $F$ as to the concept $G$

2. The extension of the concept “equinumerous to the concept $F$” is equal to the extension of the concept “equinumerous to the concept $G$”

(Logical Proposition)

(FA §69)
“Only with the definite article or a demonstrative pronoun does it [a concept word] function as a proper name of a thing, but it then ceases to function as a concept word.”

(FA, §51)
Definition of the Cardinal Number 0

The number 0 is the cardinal number which belongs to the concept “not identical with itself.”

The number 0 is the extension of the concept “equinumerous to the concept not identical with itself”

(FA, §74)
The number 1 is the cardinal number which belongs to the concept “identical with 0”

The number 1 is the extension of the concept “equinumerous to the concept identical with 0”

(FA, §77)
Frege assumed that:

1. Concepts of number and successor relation could be logically formulated and the individual numbers are logical objects (Proved)
2. Extension of a concept is logically unproblematic.
3. Extensions of concepts are objects.
4. Every concept must be defined for all objects.

By these assumptions he has shown that: Arithmetic can be derived from logic.
According to Frege:
In the sentence ‘The concept horse is a concept easily attained’, ‘the concept horse’ denotes an object because of the definite article. It is an object that falls under the concept “… is a concept easily attained”.

Frege, “Concept and Object” (CO),
In *The Frege Reader*, (Michael Beaney, Ed.), 1997: 184

So, “the concept horse is not a concept”

(CO, p. 185)

“It must be indeed be recognized that here we are confronted by an awkwardness of language, which I admit cannot be avoided, if we say that the concept horse is not a concept, whereas, e. g. the city of Berlin is a city, and the volcano Vesuvius is a volcano. Language here is in a predicament that justifies the departure from custom.”

(CO, p. 185)
It is not the awkwardness of language that appears in there but a trouble in his theory, since if he cannot use the definite article in his defined sense he does not have a tool to produce singular terms linguistically that denotes recognizable individual logical objects such as numbers.

This indicates that he cannot define numbers as logical objects. Without classifying number concept words as object words, none of his claims could run in his thesis.
Frege, by trying to free arithmetic from intuitions, tried to define numbers as logical objects by disregarding Kant’s insistence on transcendental logic.

According to Kant, mathematical objects cannot be devoid of intuitions and these objects should be constructed by the help of pure concepts of the Understanding (categories), transcendental schema and pure intuitions (space and time).

For this reason general logic is not enough to construct objects of mathematics according to Kant.
Passageway to intuitions

Frege tried to refute this claim for objects of arithmetic and tried to give the definition of individual numbers as logical objects. When Frege’s explanation on constructing logical objects is undermined, his refutation of requirement of intuitions in arithmetic is also challenged.

This opens a passageway to intuitions to define the basic mathematical objects of arithmetic.
Immediate and singular

Frege objects immediacy, since it is not communicable, and cannot be logically formulated. Hence it is of no benefit to arithmetic and mathematics.
Logical *explanation* for the immediacy can be provided where the immediacy of an intuition is interpreted as exhibiting the object *a priori* corresponding to the concept.
Frege’s definition of number is purely logical, whereas Kant’s formulation is cognitive. Kant did not have the logical tools Frege invented...
When one has the empirical intuition of 3 houses, the judgment is “There are three houses”. The Quantity of the Judgment is Particular and in the Categories this corresponds to Plurality.

In order for one to hold this unity of Judgment of Particular Quantity, one needs the unity of synthetic representations through time, which is given by apperception.
The form of time, with its unity given by apperception, as a successor relation, makes grounds possible for cognizing plurality, where plurality as a sub category provides the a priori ground for exhibiting the intuition corresponding to that concept of number 3. Then combined with the empirical intuition of 3 houses the judgment “There are 3 houses” arises.
Frege on numbers
An Example:
“The number 3”

- The number 0 is the cardinal number which belongs to the concept “not identical with itself.”

- The number 1 is the cardinal number which belongs to the concept “identical with 0”

- The number 2 is the cardinal number which belongs to the concept “identical with 0 or 1”

- The number 3 is the cardinal number that belongs to the concept “member of the natural number series ending with 2”
Frege’s theory applied to intuitions

What Frege says logically is a good way to formulate our intuitions of numbers. When one thinks about the concept “not identical with itself”, it is clear that it is *a priori* and it is *logical*. Although one cannot have the empirical intuition of 0 objects, from this concept one can refer to the concept of number 0.
Thank you
Questions welcome now or email:
ozge.ekin@gmail.com